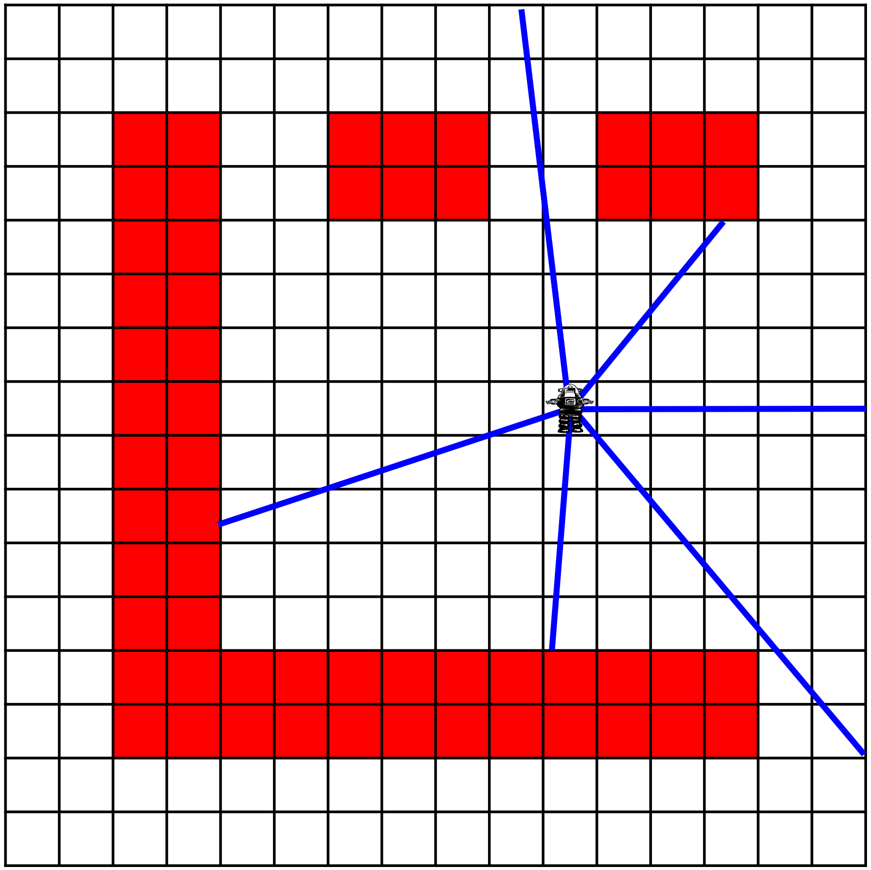
Day 33

Range Sensor Models

Beam Models of Range Finders

- given the map and the robot's location, find the probability density that the range finder detects an object at a distance z_t^k along a beam

$$p(z_t^k \mid x_t, m)$$



the k is here because range sensors typically return many measurements at one time; e.g., the sensor might return a full 360 degree scan made up of K measurements at once

$$z_t = \left\{ z_t^1, z_t^2, \ldots, z_t^k, \ldots, z_t^K \right\}$$

Beam Models of Range Finders

- most range finders have a minimum and maximum range
- we seek a model that can represent
  1. the correct range measurement with noise
  2. unexpected obstacles
  3. failures
  4. random measurements

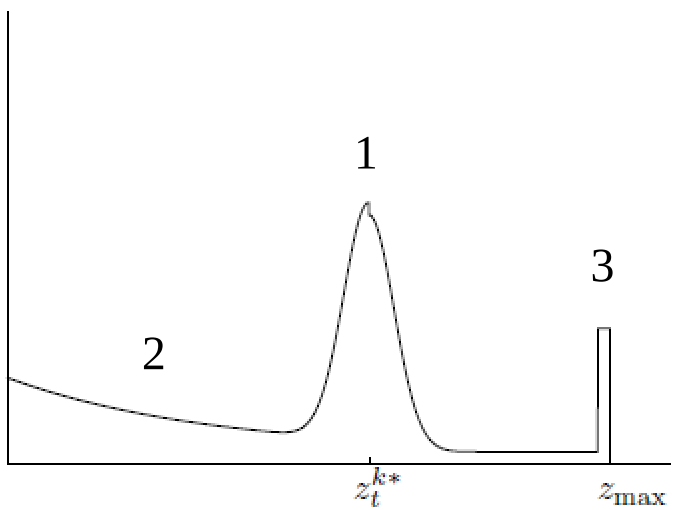


Figure 6.4 "Pseudo-density" of a typical mixture distribution $p(z_t^k \mid x_t, m)$.

Beam Models of Range Finders

- beam-based model is computationally expensive
  - each measurement requires a ray intersection with the map to compute z_t^{k*}

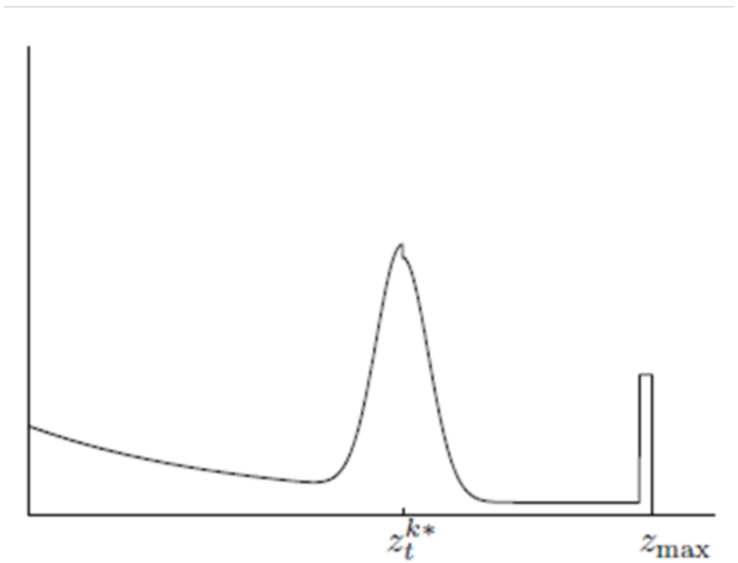
Velodyne HDL-64E S2
>1.3 million measurements per second

Beam Models of Range Finders

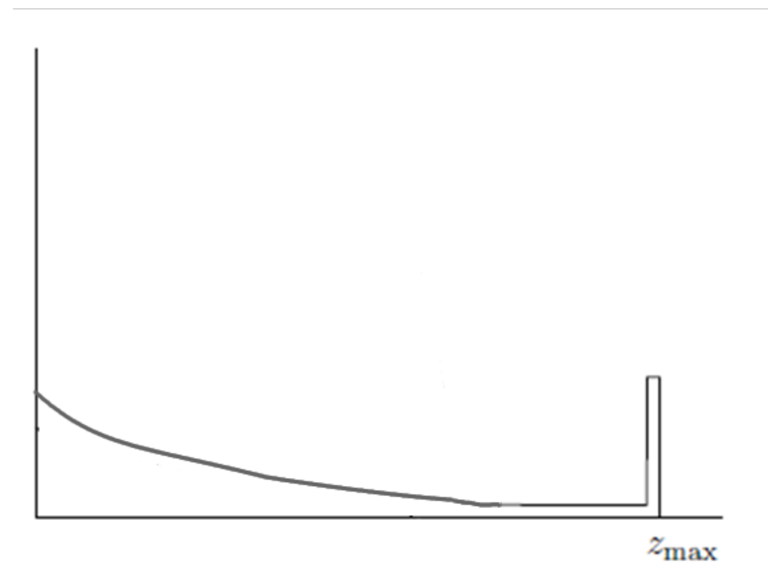
- computational expense can be reduced by precomputing z_t^{k*}
  - decompose state space into a 3D grid
    - much like histogram filter
  - for each grid location compute and store z_t^{k*}
- much faster than ray casting but…

Beam Models of Range Finders

- $p(z_t^k \mid x_t, m)$ changes abruptly for small changes in x_t
  - especially true for small changes in θ



map obstacle detected for robot bearing θ

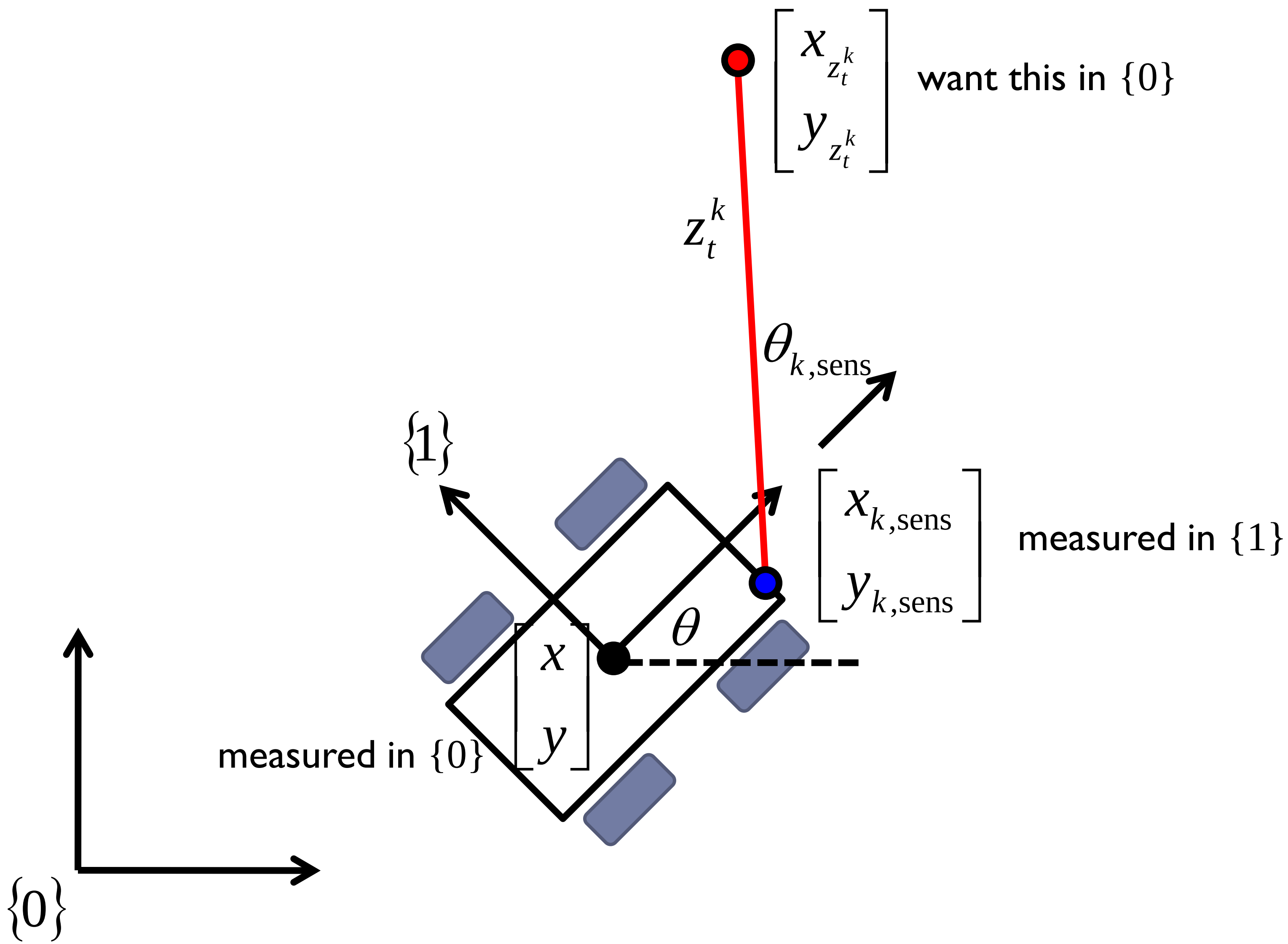


map obstacle missed for robot bearing $\theta + d\,\theta$

Likelihood Fields for Range Finders

- an alternative model called *likelihood fields* overcomes the limitations of the beam model
  - but is not physically meaningful
- recall that a range measurement z_t^k is a distance measured relative to the robot
  - can we transform z_t^k into a point in the map?

Likelihood Fields for Range Finders



Likelihood Fields for Range Finders

$$\begin{bmatrix} x_{z_t^k} \\ y_{z_t^k} \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_{k,\text{sens}} \\ y_{k,\text{sens}} \end{bmatrix} + z_t^k \begin{bmatrix} \cos(\theta + \theta_{k,\text{sens}}) \\ \sin(\theta + \theta_{k,\text{sens}}) \end{bmatrix}$$

Likelihood Fields for Range Finders

- similar to the beam model, we assume 3 types of sources of noise and uncertainty

1. correct measurement with noise
2. failures
3. random measurements

Likelihood Fields for Range Finders

1. Correct Measurement with Noise

- once we have $[x_{z_t^k} \quad y_{z_t^k}]^T$ we find the point on an obstacle in the map that is nearest to $[x_{z_t^k} \quad y_{z_t^k}]^T$ and compute the distance d between those two points
- this measurement is modeled as a zero-mean Gaussian with variance σ_{hit}^2

$$p_{\text{hit}}(z_t^k \mid x_t, m) = \varepsilon_{\sigma_{\text{hit}}^2}\left(d \right)$$

Likelihood Fields for Range Finders

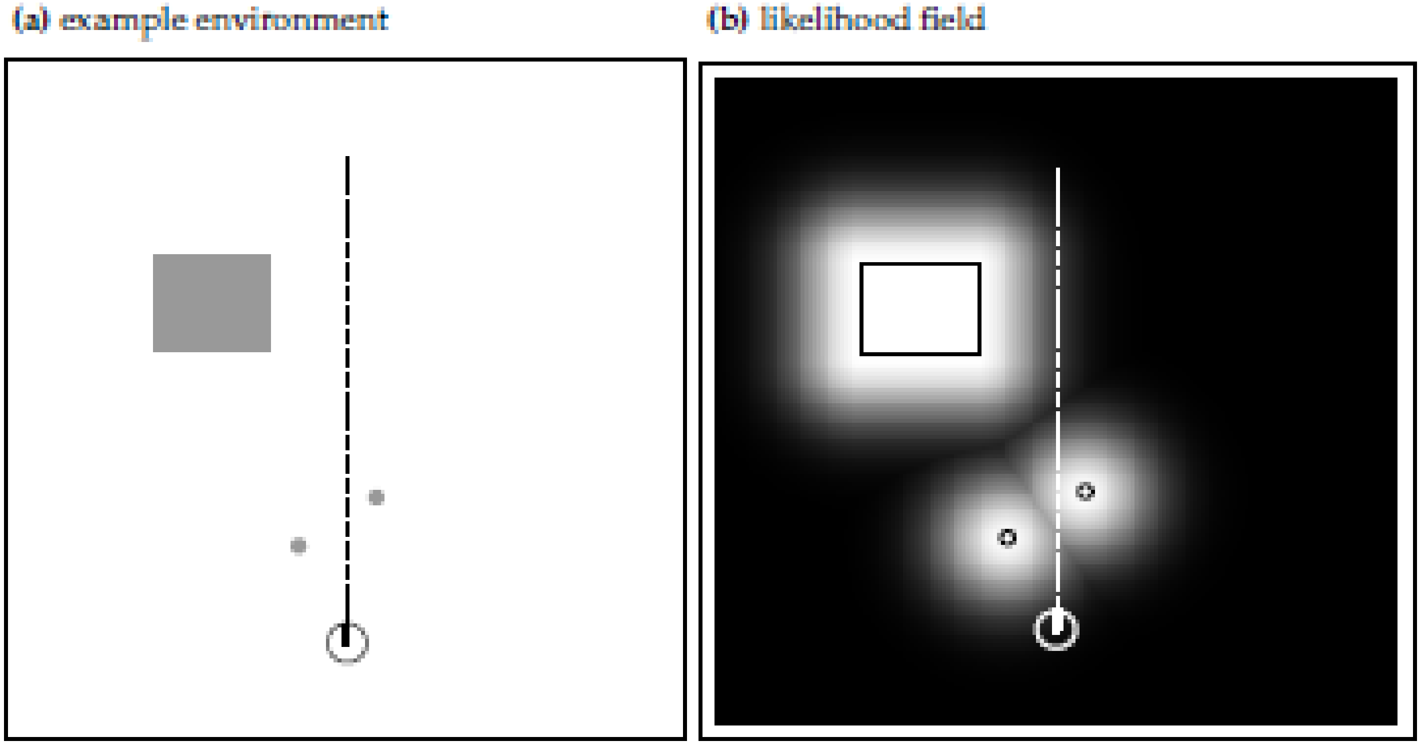


Figure 6.8 (a) Example environment with three obstacles (gray). The robot is located towards the bottom of the figure, and takes a measurement z_t^k as indicated by the dashed line. (b) Likelihood field for this obstacle configuration: the darker a location, the less likely it is to perceive an obstacle there. The probability $p(z_t^k \mid x_t, m)$ for the specific sensor beam is shown in Figure 6.9.

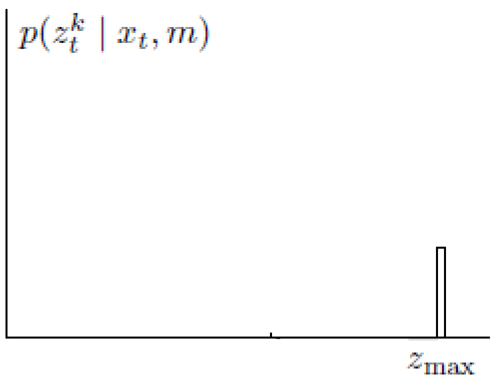
Beam Models of Range Finders

2. Failures

- range finders can fail to sense an obstacle in which case most sensors return $z_{\max}$
- modeled as a point mass distribution centered at $z_{\max}$

$$p_{\max}(z_t^k \mid x_t, m) = \begin{cases} 1 & \text{if } z_t^k = z_{\max} \\ 0 & \text{otherwise} \end{cases}$$

(c) Uniform distribution $p_{\max}$

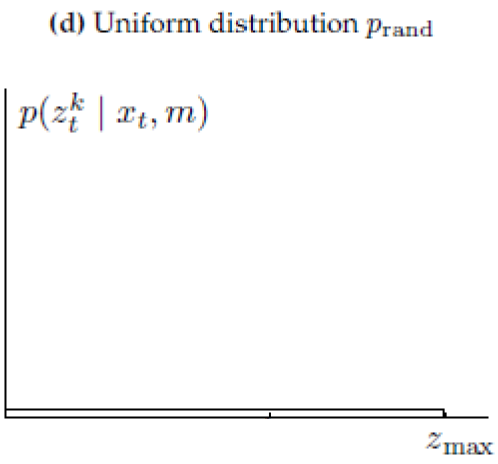


Beam Models of Range Finders

3. Random Measurements

- unexplainable measurements are modeled as a uniform distribution over the range $[0, z_{\max}]$

$$p_{\text{rand}}(z_t^k \mid x_t, m) = \begin{cases} \frac{1}{z_{\max}} & \text{if } 0 \le z_t^k \le z_{\max} \\ 0 & \text{otherwise} \end{cases}$$



Beam Models of Range Finders

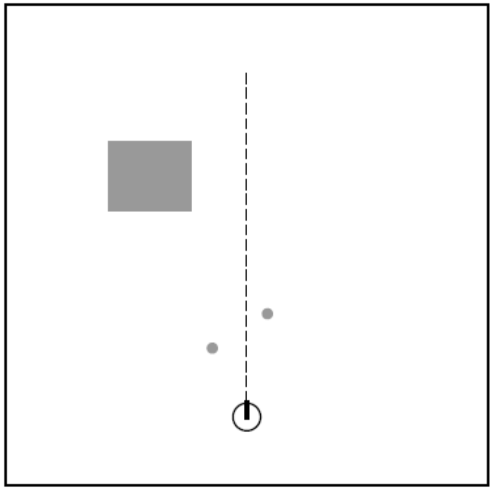
- the complete model is a weighted sum of the previous three densities with weights

$$w_{\text{hit}} + w_{\text{max}} + w_{\text{rand}} = 1$$

$$p(z_t^k \mid x_t, m) = \begin{bmatrix} w_{\text{hit}} \\ w_{\text{max}} \\ w_{\text{rand}} \end{bmatrix}^T \begin{bmatrix} p_{\text{hit}}(z_t^k \mid x_t, m) \\ p_{\text{max}}(z_t^k \mid x_t, m) \\ p_{\text{rand}}(z_t^k \mid x_t, m) \end{bmatrix}$$

Beam Models of Range Finders

(a) example environment



(a) $p_{\text{hit}}(z_t^k \mid x_t, m)$ (b) $p(z_t^k \mid x_t, m)$

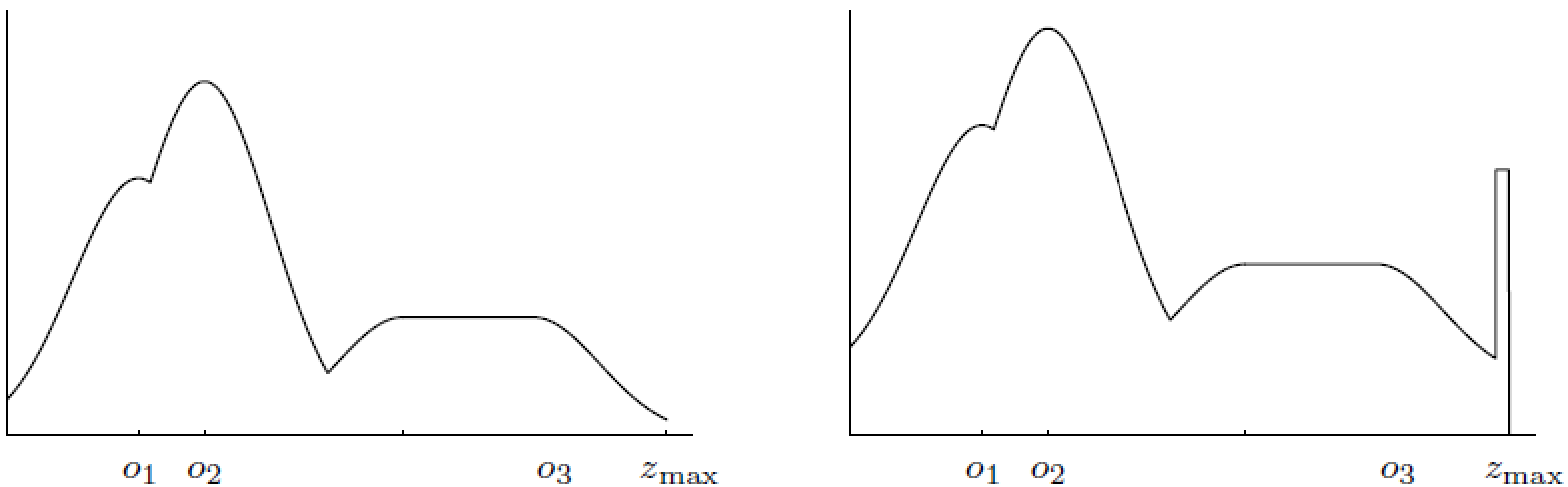


Figure 6.9 (a) Probability $p_{\text{hit}}(z_t^k)$ as a function of the measurement z_t^k, for the situation depicted in Figure 6.8. Here the sensor beam passes by three obstacles, with respective nearest points o_1, o_2, and o_3. (b) Sensor probability $p(z_t^k \mid x_t, m)$, obtained for the situation depicted in Figure 6.8, obtained by adding two uniform distributions.